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# Polar-optical-phonon and electron–electron scattering in large-bandgap semiconductors

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**Abstract.** The low-field electron mobility associated with polar-optical-phonon scattering in GaN is calculated from an exact solution of the Boltzmann equation at various electron densities up to $10^{19}$ cm$^{-3}$. At low electron densities the mobility = 2200 cm$^2$ V$^{-1}$ s$^{-1}$ ($m^* = 0.22\ m_0$) or 2500 cm$^2$ V$^{-1}$ s$^{-1}$ ($m^* = 0.20\ m_0$), and it decreases with increasing electron density. The standard analytical model for the mobility is found to be reasonably accurate in the non-degenerate regime. The effect of electron–electron scattering is discussed. A new expression for the carrier–carrier scattering rate which embodies the binary nature of the process is advanced and used to estimate the effect of electron–electron scattering on the polar-optical-phonon mobility. Our estimate suggests that electron–electron scattering roughly cancels out the drop in mobility leaving the mobility largely independent of electron density.

## 1. Introduction

Scattering of carriers by optical phonons is the principal mechanism determining mobility at room temperature in pure large-bandgap semiconductors, and consequently it has received a good deal of attention [1–3], particularly in the context of describing mobility in that paradigm semiconductor, GaAs [4–8]. More recently, interest has focused on GaN and associated compounds and estimates of room-temperature electron mobility have been made based on a conveniently simple analytical model—the standard model—which describes polar-optical-mode scattering in terms of a relaxation time determined by the absorption of phonons [9]. This model also gives a good account of the mobility in GaAs, but then so does an equally simple model of Callen's [2] which is exact when the distribution function is a drifted Maxwellian (or Fermi–Dirac). A comparison of the standard model and the drifted model in the case of GaN showed that there was a large discrepancy between the magnitudes of the mobilities predicted and that the standard model was the more valid [10], at least for low electron densities. The existence of the drifted model and its prediction of larger mobilities serves, nevertheless, to raise the problem of the effect of electron–electron scattering on polar-optical-phonon determined mobility. The whole question of how the mobility is affected by increasing electron density is particularly relevant to the case in GaN, and its related compounds, whose strong piezoelectricity can be used to introduce large populations of electrons without doping [11]. An exact solution of the Boltzmann equation in the absence of electron–electron scattering but taking into account degeneracy predicted that the mobility falls significantly with increasing density [12]. Assessing the effect of electron–electron scattering is therefore of some interest, and we discuss this in the present paper.

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The effects of carrier–carrier scattering on the mobility in semiconductors have been considered by a number of people [13–19]. Electron–electron (or hole–hole) scattering usually produces only minor changes in mobility. An exception is when the low-temperature mobility is determined by charged-impurity scattering, in which case the mobility can be reduced by as much as 40% [15]. The effect depends upon the relaxation time associated with the dominant scattering mechanism being energy dependent: the more energy dependent the bigger the effect. However, the maximum change possible is set by the distribution function becoming a drifted Maxwellian or Fermi–Dirac function. In the case of GaAs the effect on the polar-optical mobility is predicted to be small because the standard and drifted models give almost the same mobility. But this is not the case for the polar-optical mobility in GaN, where the drifted mobility is over three times greater than that of the standard model. Large effects of electron–electron scattering can therefore be expected, and in what follows we attempt to estimate these in the case of pure bulk GaN at room temperature.

## 2. The Boltzmann equation

Much of the early work on electron–electron scattering derived from the theory of ionized gases [13, 20], but this work emphasized the importance of distant collisions and described the effects in terms of the Fokker–Planck equation. In solids these distant interactions become implicitly involved, in time-averaged form, in the potential an electron experiences in its motion through the lattice as a Bloch wave. Encounters with other electrons can therefore be regarded as comparatively rare and predominantly of the binary type. It is thus appropriate to treat electron–electron collisions in the same way as other scattering events, by the Boltzmann equation.

In the presence of a weak electric field $\boldsymbol{F}$ the Boltzmann equation can be linearized by taking the distribution function to be of the form:

$$f(\boldsymbol{k}) = f_0(E) + \frac{\mathrm{d}f_0(E)}{\mathrm{d}E}\Phi(\boldsymbol{k})\cdot\boldsymbol{F} \tag{1}$$

and neglecting quadratic terms. Here, $\boldsymbol{k}$ is the wavevector and $E$ is the energy of the electron. The Boltzmann equation is:

$$e\boldsymbol{F}\cdot\boldsymbol{v}(\boldsymbol{k})\frac{\mathrm{d}f_0(E)}{\mathrm{d}E} = \left[\frac{\mathrm{d}f(\boldsymbol{k})}{\mathrm{d}t}\right]_{po} + \left(\frac{\mathrm{d}f(\boldsymbol{k})}{\mathrm{d}t}\right)_{ee} \tag{2}$$

where $\boldsymbol{v}(\boldsymbol{k})$ is the group velocity of the electron. We consider only polar-optical-phonon scattering and electron–electron scattering within a spherically symmetric, parabolic conduction band.

The scattering components can be expressed in standard form. The rate for polar-optical-phonon scattering is given by:

$$\left[\frac{\mathrm{d}f(\boldsymbol{k})}{\mathrm{d}t}\right]_{po} = \frac{\boldsymbol{F}}{\boldsymbol{k_B T}}\cdot\int V(\boldsymbol{k},\boldsymbol{k}')[\Phi(\boldsymbol{k}')-\Phi(\boldsymbol{k})]\,\mathrm{d}\boldsymbol{k}'\frac{\Omega}{(2\pi)^3}$$

$$V(\boldsymbol{k},\boldsymbol{k}') = W(\boldsymbol{k},\boldsymbol{k}')f_0(E)(1-f_0(E))$$

$$W(\boldsymbol{k},\boldsymbol{k}') = W_0\frac{2\pi}{\Omega}\left(\frac{\overline{h}^3\omega_{LO}}{2m^*}\right)^{1/2}\delta_{\boldsymbol{k}',\boldsymbol{k}+\boldsymbol{q}}\frac{1}{q^2}(n(\omega_{LO})+\tfrac{1}{2}\pm\tfrac{1}{2})\delta(E'-E\pm\overline{h}\omega_{LO})$$

$$W_0 = \frac{e^2}{4\pi\overline{h}}\left(\frac{2m^*\omega_{LO}}{\overline{h}}\right)^{1/2}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_s}\right). \tag{3}$$

Here, $T$ is the absolute temperature, $\Omega$ is the cavity volume, $\omega_{LO}$ is the LO phonon frequency, $n(\omega_{LO})$ is the phonon occupation factor phonon, $q$ is the wavevector and $\varepsilon_\infty$, $\varepsilon_s$ are the high-frequency and low-frequency permittivities. The rate for electron–electron scattering is:

$$\left(\frac{\mathrm{d}f(\boldsymbol{k}_1)}{\mathrm{d}t}\right)_{ee} = \frac{\boldsymbol{F}}{\boldsymbol{k_B T}} \cdot \int V(\boldsymbol{k}_1'\boldsymbol{k}_2'\boldsymbol{k}_1\boldsymbol{k}_2)[\Phi(\boldsymbol{k}_1') + \Phi(\boldsymbol{k}_2') - \Phi(\boldsymbol{k}_1) - \Phi(\boldsymbol{k}_2)] \times \mathrm{d}\boldsymbol{k}_1' \,\mathrm{d}\boldsymbol{k}_2 \,\mathrm{d}\boldsymbol{k}_2' \frac{2\Omega^3}{(2\pi)^9}$$

$$V(\boldsymbol{k}_1'\boldsymbol{k}_2'\boldsymbol{k}_1\boldsymbol{k}_2) = W(\boldsymbol{k}_1'\boldsymbol{k}_2'\boldsymbol{k}_1\boldsymbol{k}_2) f_0(E_1') f_0(E_2')(1 - f_0(E_1))(1 - f_0(E_2))$$
$$= W(\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_1'\boldsymbol{k}_2') f_0(E_1) f_0(E_2)(1 - f_0(E_1'))(1 - f_0(E_2'))$$

$$W(\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_1'\boldsymbol{k}_2') = \frac{2\pi}{\hbar}|M|^2 \delta(E_1' + E_2' - E_1 - E_2)$$

$$|M|^2 = |M_{12}|^2 + |M_{21}|^2 - |M_{12}||M_{21}|$$

$$M_{12} = \frac{e^2}{\Omega\varepsilon(q,\omega)} \frac{1}{|\boldsymbol{g}_{12} - \boldsymbol{g}_{12}'|^2} \delta_{\boldsymbol{k}_1' + \boldsymbol{k}_2', \boldsymbol{k}_1 + \boldsymbol{k}_2}. \tag{4}$$

In this $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are the wavevectors of the incident and struck electrons and $\boldsymbol{k}_1'$, $\boldsymbol{k}_2'$ are the wavevectors after the collision. The matrix element takes into account exchange and interference between like spins. The wavevectors $\boldsymbol{g}_{12}$ and $\boldsymbol{g}_{12}$' are the relative wavevectors before and after the collision i.e.:

$$\boldsymbol{g}_{12} = \tfrac{1}{2}(\boldsymbol{k}_1 - \boldsymbol{k}_2) \qquad \boldsymbol{g}_{12}' = \tfrac{1}{2}(\boldsymbol{k}_1' - \boldsymbol{k}_2'). \tag{5}$$

The quantity $\varepsilon(q,\omega)$ is the appropriate dielectric function for the collision [21]

$$\varepsilon(q,\omega) = \varepsilon_L(\omega) + \varepsilon_e(q,\omega). \tag{6}$$

The lattice permittivity is:

$$\varepsilon_L(\omega) = \varepsilon_\infty \frac{\omega_{LO}^2 - \omega^2}{\omega_{TO}^2 - \omega^2} \tag{7}$$

and the electron contribution is given in the random-phase approximation by the Lindhard function:

$$\varepsilon_e(q,\omega) = \frac{e^2}{q^2\Omega} \sum_{\boldsymbol{k}} f(E_{\boldsymbol{k}}) \left[ \frac{1}{E_{\boldsymbol{k}-\boldsymbol{q}} - E_{\boldsymbol{k}} + \hbar\omega + \mathrm{i}\hbar\alpha} + \frac{1}{E_{\boldsymbol{k}+\boldsymbol{q}} - E_{\boldsymbol{k}} - \hbar\omega - \mathrm{i}\hbar\alpha} \right]. \tag{8}$$

The relevant frequency here is $\boldsymbol{q} \cdot \boldsymbol{v}_{cm}$, where $q$ is $\boldsymbol{g}_{12} - \boldsymbol{g}_{12}'$; $\boldsymbol{v}_{cm}$ is the velocity of the centre of mass, $(\boldsymbol{v}_1 + \boldsymbol{v}_2)/2$.

A solution to the Boltzmann equation is usually sought via the employment of variational [15, 16, 25] or iterative [6] techniques with screening simplified by ignoring dynamic aspects. These techniques are most successful when the vector $\phi(\boldsymbol{k})$ defining the antisymmetric part of the distribution function is well behaved and has no sudden discontinuities. Unfortunately, this is not the case for optical-phonon scattering, where a sudden onset of phonon emission occurs when $E = \hbar\omega_{LO}$. Discontinuities of this sort are also going to affect the ability of electron–electron scattering to randomize the momentum gained from the field and produce a drifted distribution. It seems promising, therefore, to focus on the phonon-emission discontinuity and to allow its strength to determine the effect electron–electron scattering can have.

## 3. Exact solution for polar-optical-phonon scattering only

We first use an (in principle) exact method for solving the Boltzmann equation in the absence of electron–electron scattering in order to define the distribution function and its discontinuities. The method, similar to that used by Delves [22], is fully described by Fletcher and Butcher [8], who used it for n-GaAs. Only an outline of the method is given here.

The vector $\phi(\boldsymbol{k})$ can be expressed in terms of $\boldsymbol{k}$ and a scalar function of energy $\tau(E)$, which can be identified as an effective relaxation time, *viz.*

$$\phi(\boldsymbol{k}) = \frac{e\bar{h}}{m^*}\tau(E)\boldsymbol{k}. \tag{9}$$

The field can be eliminated from (2) and the integrations performed in the standard way to give the equation:

$$ZE^{3/2} = -A(E)\tau(E+\bar{h}\omega_{LO}) - B(E)\tau(E-\bar{h}\omega_{LO}) + C(E)\tau(E)$$

$$A(E) = (n(\omega_{LO})+1)\frac{f_0(E+\bar{h}\omega_{LO})}{f_0(E)} \times \left\{(2E+\bar{h}\omega_{LO})\sinh^{-1}\left(\frac{E}{\bar{h}\omega_{LO}}\right)^{1/2} - [E(E+\bar{h}\omega_{LO})]^{1/2}\right\}$$

$$B(E) = \theta(E-\bar{h}\omega_{LO})n(\omega_{LO})\frac{f_0(E-\bar{h}\omega_{LO})}{f_0(E)} \times \left\{(2E-\bar{h}\omega_{LO})\cosh^{-1}\left(\frac{E}{\bar{h}\omega_{LO}}\right)^{1/2} - [E(E-\bar{h}\omega_{LO})]^{1/2}\right\}$$

$$C(E) = 2E\left[(n(\omega_{LO}+1)\frac{f_0(E+\bar{h}\omega_{LO})}{f_0(E)}\sinh^{-1}\left(\frac{E}{\bar{h}\omega_{LO}}\right)^{1/2} + \theta(E-\bar{h}\omega_{LO})n(\omega_{LO})\frac{f_0(E-\bar{h}\omega_{LO})}{f_0(E)}\cosh^{-1}\left(\frac{E}{\bar{h}\omega_{LO}}\right)^{1/2}\right]$$

$$Z = \frac{2}{W_0(\bar{h}\omega_{LO})^{1/2}} \tag{10}$$

$\theta(x)$ is the step function. If $\tau(E)$ were known in the interval $0 < E < \bar{h}\omega_{LO}$, (10) would generate $\tau(E)$ in all steps of the phonon ladder. Unfortunately, small errors become rapidly gigantic in this procedure, so guessing $\tau(E)$ is not viable. However, we know the form of $\tau(E)$ when $E \gg \bar{h}\omega$, *viz.*

$$\tau(E) \to \frac{ZE^{3/2}}{C(E) - A(E) - B(E)}. \tag{11}$$

Thus we can approximate $\tau(E+\bar{h}\omega)$ for large $E$ to the expression in (11) and work down the ladder in order to obtain accurate values for $\tau(E)$, $E < \bar{h}\omega$. Details of the method can be found in the paper by Fletcher and Butcher [8].

This method has been applied to GaN [12] to yield $\tau(E)$ over the first three rungs of the phonon ladder. Since for GaN $3\bar{h}\omega_{LO} \gg k_BT$ at 300 K, three rungs were sufficient even for moderate degeneracy. Results for electron densities up to $10^{19}$ cm$^{-3}$ are shown in figure 1 along with a comparison with the two analytical models. The relaxation times for these models were given by:

$$\text{standard}: \tau(E) = \frac{ZE^{3/2}}{C(E)} \qquad E < \bar{h}\omega_{LO}$$

$$\text{drifted} : \tau(E) = \frac{ZE^{3/2}}{C(E) - A(E) - B(E)}. \tag{12}$$

The calculations used the following magnitudes: $m^* = 0.22\ m_0$, $\hbar\omega_{LO} = 0.092$ eV, $\varepsilon_s = 9.0\ \varepsilon_0$, $\varepsilon_\infty = 5.37\ \varepsilon_0$, $T = 300$ K. The standard model turns out to be an excellent approximation, whereas the drifted model predicts relaxation times several times too large. The discontinuity at $E = \hbar\omega_{LO}$ is large at all levels of degeneracy investigated. The effect of degeneracy is to reduce the time-constants. The effects of exclusion, which would tend to increase the relaxation time, are more than compensated by the increase in number of electron capable of emission. As noted previously this effect results in a marked drop of mobility with increasing electron concentration [12].

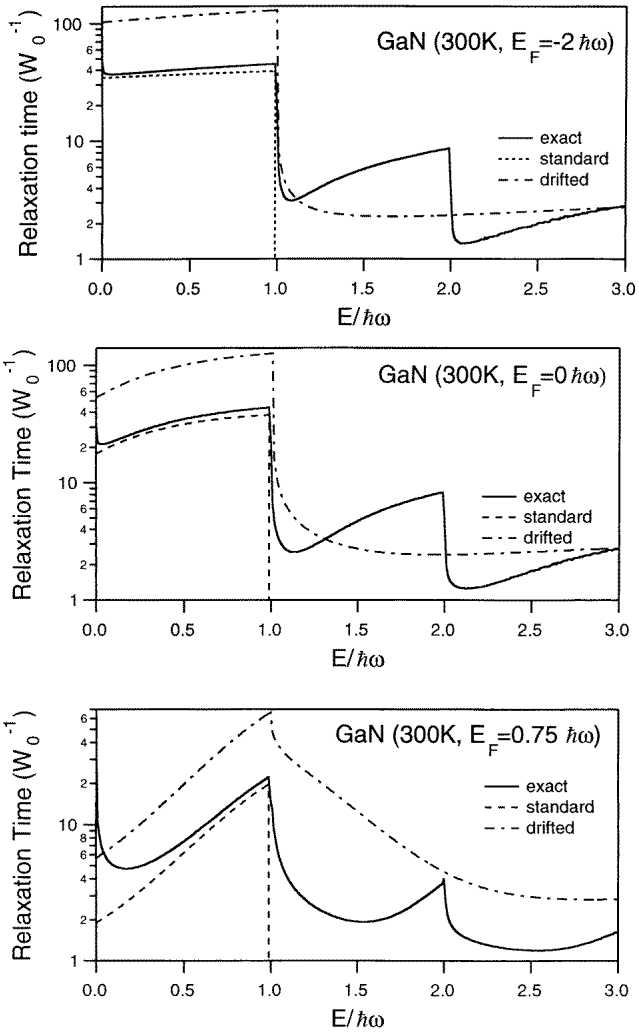


**Figure 1.** Momentum-relaxation times for polar-optical-phonon scattering of electrons in GaN at 300 K as calculated from an exact solution of the Boltzmann equation, and compared to the predictions of the standard model (absorption only) and the drifted model (drifted Fermi–Dirac distribution), for three values of the Fermi level.

The longer relaxation time predicted by the drifted model suggests that electron–electron scattering will counteract that drop in mobility. We now turn to an estimate of the magnitude of this effect.

## 4. Electron–electron scattering

Of the nine integrals in (4) three, those over $\boldsymbol{k}_2'$, yield unity as a result of momentum conservation. The integrals over $\boldsymbol{k}_1'$ can be transformed to those over $\boldsymbol{g}_{12}'$, since $\boldsymbol{k}_1' = \boldsymbol{g}_{12}' + \boldsymbol{k}_{cm}$. Noting that:

$$E_1' + E_2' - E_1 - E_2 = \frac{\hbar^2}{2m^*}(k_1'^2 + k_2'^2 - k_1^2 - k_2^2) = \frac{\hbar^2}{m^*}(g_{12}'^2 - g_{12}^2) \tag{13}$$

we can integrate over $g_{12}'$ with $\boldsymbol{g}_{12}$ as the polar axis, and obtain

$$\left(\frac{\mathrm{d}f(\boldsymbol{k}_1)}{\mathrm{d}t}\right)_{ee} = \frac{e\pi}{\hbar^2 k_B T}\boldsymbol{F}\cdot\int |M|^2 G(E)[\tau(E_1')\boldsymbol{k}_1' + \tau(E_1)\boldsymbol{k}_2' - \tau(E_1)\boldsymbol{k}_1 - \tau(E_2)\boldsymbol{k}_2]$$
$$\times \mathrm{d}(-\cos\phi)\,\mathrm{d}\alpha_{12}\,\mathrm{d}\boldsymbol{k}_2\frac{2\Omega^2}{(2\pi)^6}$$
$$G(E) = f_0(E_1')f_0(E_2')(1 - f_0(E_1))(1 - f_0(E_2))$$
$$|M|^2 = \left(\frac{e^2}{\Omega}\right)^2 \frac{1}{|\varepsilon(q,\omega)|^2}\left\{\frac{1}{(2g_{12}\sin(\phi/2))^4} + \frac{1}{(2g_{21}\cos(\phi/2))^4}\right.$$
$$\left. - \frac{1}{(2g_{12}\sin(\phi/2))^2(2g_{21}\cos(\phi/2))^2}\right\}. \tag{14}$$

$E_1'$ and $E_2'$ depend upon $\phi$, the angle between $\boldsymbol{g}_{12}$ and $\boldsymbol{g}_{12}'$. Although $f_0(E_1')$ and $f_0(E_2')$ are known by our assumption that the spherically symmetric part of the distribution function is Fermi–Dirac at the lattice temperature, the relaxation times $\tau(E_1')$ and $\tau(E_2')$ are unknowns, so further progress can be made only by resorting to laborious variational or iterative techniques, or by making approximations. In this paper the latter approach is adopted.

Before going any further, we need to make a general comment on the derivation of the electron–electron scattering rate as given by (4). It is a comment that has been made before in the context of charged-impurity scattering [23, 24] and it applies equally strongly here. The essential assumption is that the scattering process is binary, but because the interaction is coulombic, infinitely large cross-sections associated with distant interaction are predicted. In (14), for example, the rate is inversely proportional to terms such as $(2g_{12}\sin\phi_{12})^4$ which can be zero. Embarrassment is avoided usually by invoking static screening:

$$\varepsilon(q,\omega) = \varepsilon_s(1 + q_0^2/q^2) \tag{15}$$

where $q_0$ is reciprocal screening length, but in general this is not valid since screening is essentially dynamic [21] and may become negligible in some cases. In order to avoid infinite cross-section it is necessary to truly restrict collisions to binary encounters by weighting the rate with probability that no nearer scatterer exists, thus limiting the rate to that associated with collisions with nearest neighbours. The probability $P(b)$ that no scattering centre exists with impact parameter less than $b$ is given by

$$P(b) = \exp(-\pi N a b^2)$$
$$b = b_0\cot(\phi/2) \tag{16}$$

where $N$ is the density of scattering centres, $a$ is the average separation, given by $(2\pi N)^{-1/3}$, and $\phi$ is the scattering angle. We propose, therefore, to modify the squared matrix element in (14) as follows:

$$|M|^2 = \left(\frac{e^2}{\Omega}\right)^2 \frac{1}{|\varepsilon(q,\omega)|^2}\left\{\frac{P(b_{12})}{[2g_{12}\sin(\phi/2)]^4} + \frac{P(b_{21})}{[2g_{21}\cos(\phi/2)]^4} - \frac{P(b_{12})P(b_{21})}{[2g_{12}\sin(\phi/2)2g_{21}\cos(\phi/2)]^2}\right\} \tag{17}$$

where

$$P(b_{12}) = \exp\left[-\left(\frac{\pi N}{4}\right)^{2/3}\left[\frac{e^2}{4\pi|\varepsilon(q,\omega)|}\right]^2 \frac{m^{*2}}{4\overline{h}^4 g_{12}^4}\cot(\phi/2)\right]$$
$$P(b_{21}) = \exp\left[-\left(\frac{\pi N}{4}\right)^{2/3}\left[\frac{e^2}{4\pi|\varepsilon(q,\omega)|}\right]^2 \frac{m^*2}{4\overline{h}^4 g_{21}^4}\tan(\phi/2)\right]. \tag{18}$$

With this modification, electron–electron scattering rates remain properly binary-type rates.

We can now return to the problem of how electron–electron scattering affects polar-optical-phonon mobility. The following comments can be made:

(1) In a fully drifted distribution $\tau(E_1') = \tau(E_2') = \tau(E_1) = \tau(E_2)$, and hence the net rate vanishes.

(2) The occupation factor $G(E)$ favours collisions involving electrons within about $k_BT$ of the bandedge, in the non-degenerate case, or of the Fermi level in the degenerate case. Unless $\tau(E)$ is a strong function of energy near the bandedge or near the Fermi level, the net electron–electron rate for these most probable events will be small. Charged-impurity scattering in a non-degenerate population at low temperatures does indeed have a relaxation time that varies relatively rapidly with energy, and that is why electron–electron scattering has a significant effect in that case [15]. In the case of other elastic or quasi-elastic scattering processes the variation with energy of $\tau(E)$ is rather weak and, consequently, electron–electron scattering has little effect.

(3) We notice that in the case of optical-phonon scattering the dependence of $\tau(E)$ on energy is relatively weak except at the discontinuities. Thus for non-degenerate and weakly degenerate material the most frequent electron–electron collisions can have only a correspondingly weak effect on the mobility. It follows that any significant effect on the mobility can come only from less frequent collisions involving an electron at or just above the first discontinuity.

We therefore take as a measure of the strength of electron-electron scattering to affect the mobility the ratio of the electron–electron scattering-out rate $W_{ee}$ at $E = \overline{h}\omega_{LO}$ to the characteristic phonon rate $W_0$. If $\mu_{po}$ is the mobility in the absence of electron–electron scattering and $\mu_{drift}$ is the corresponding mobility for a drifted distribution, then we estimate the effect of electron–electron scattering via the simple formula:

$$\mu = \frac{W_0\mu_{po} + W_{ee}\mu_{drift}}{W_0 + W_{ee}}. \tag{19}$$

The calculation of $W_{ee}$ is straightforward if screening by electrons is ignored. This is a reasonable approximation since the frequency of the interaction is, for $\overline{h}^2k_1^2/2m^* = \overline{h}\omega_{LO}$ and $k_1 \gg k_2$:

$$\omega = \omega_{LO}\sin(\phi/2). \tag{20}$$

The plasmon frequency is of order $\omega_{LO}$ only when $N \approx 10^{19}$ cm$^{-3}$, so for most of the range of interest screening by the electron gas will be weak, and the dielectric function can be taken to be determined by the low-frequency response of the lattice (i.e. $\varepsilon(q, \omega) \approx \varepsilon_s$).

Using (17) for $|M^2|$ and ignoring screening allows the integrations to be straightforward. The integrations over angle $\phi$ and $a_{12}$ lead to a simple expression for the cross-section $\sigma$, namely

$$\sigma = (2\pi)^{1/3} N^{-2/3}. \tag{21}$$

The required rate is then obtained by integrating over $\boldsymbol{k}_2$. Thus

$$W_{ee} = \sigma \langle v \rangle N f_0(\bar{h}\omega_{LO}) \approx \sigma v(\bar{h}\omega_{LO}) N f_0(\bar{h}\omega_{LO}). \tag{22}$$

Inserting this rate into (19) allows us to estimate the effect of electron–electron scattering on the mobility. Figure 2 shows the result.

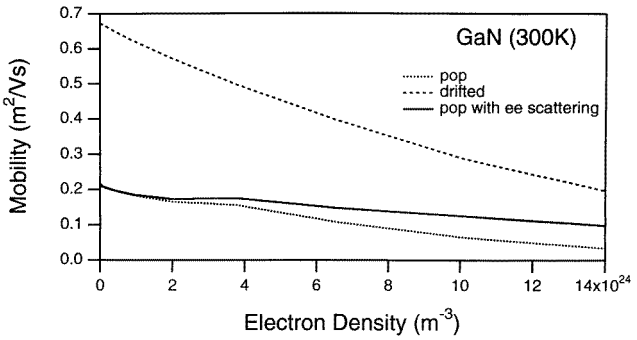


**Figure 2.** Electron mobility in GaN at 300 K as a function of electron density. The pop (polar-optical-phonon) mobility is from the exact solution without electron–electron scattering. The drifted mobility is the mobility with electron–electron scattering dominant. The pop with ee scattering is the model mobility (19).

Our model is most applicable for low levels of degeneracy. As figure 1 shows, increasing degeneracy increases the energy dependence of $\tau(E)$ away from the discontinuity, and this will tend to enhance the effect of most frequent electron–electron scattering collisions. Our result for the mobility variation at high densities therefore represents an underestimate of the effect of electron–electron scattering. This suggests that the mobility, in fact, reduces very little with electron density.

## 5. Discussion

The exact solution of the Boltzmann equation in the absence of electron–electron scattering yielded a room-temperature mobility of about 2200 cm$^2$ V$^{-1}$ s$^{-1}$. There is some uncertainty about the effective mass; if taken to be $0.20\,m_0$ the mobility would rise to 2500 cm$^2$ V$^{-1}$ s$^{-1}$. Increasing the electron concentration (without adding new scattering centres) reduces the mobility as more electrons populate states with energy $E > \bar{h}\omega_{LO}$, so allowing them to emit as well as absorb optical phonons. The appropriate analytical model for the mobility works well in the non-degenerate regime and also predicts a reduction of mobility with increasing density. Such a reduction can be expected to occur whenever $\bar{h}\omega_{LO} \gg k_B T$ and the standard model is applicable.

Increasing the electron population introduces two new effects. One is enhanced electron–electron scattering; the other is the coupling of plasmon and phonon modes. Coupled modes

broaden the spectrum of quanta with which the electron can interact and will therefore tend to soften the discontinuities in the distribution function besides varying the strength of the interaction. These effects become important when $\omega_p \cong \omega_{LO}$ where $\omega_p$ is the plasma frequency, $e^2N/\varepsilon_\infty m^*$. This occurs in GaN when $N \geqslant 10^{19}$ cm$^{-3}$. When $\omega_p^2 \gg \omega_{LO}^2$, the interaction with the phonons becomes statically screened. In this report we have ignored coupled-mode effects and instead we have focused mainly on the effect of electron–electron scattering at comparatively modest densities ($N \leqslant 10^{19}$ cm$^{-3}$).

We have contributed to the theory of electron–electron scattering by including the probability of the interaction being truly binary. This removes the divergence characteristic of the Coulomb interaction in a basic way that is independent of any details of screening. This was particularly important to do in the present context which emphasized the scattering of an electron with energy $\hbar\omega_{LO}$ by another electron since this occurred at a speed which tended to inhibit screening by the electron gas altogether.

Our treatment of the effect of electron–electron scattering on mobility has been profoundly influenced by the existence of the relaxation-time discontinuity at $E = \hbar\omega_{LO}$. By concentrating on electron–electron scattering in its vicinity rather than in the energy regime where collisions are most frequent, a choice made possible only by the comparatively large magnitude of the phonon energy, we believe we have identified the essential physics of the effect. Inevitably, given the simplicity of our model, the quantitative estimate of the change of mobility depicted in figure 2 is far from definitive. It will be interesting to compare it with the results of a more sophisticated calculation when this becomes available. However, our estimate suggests that electron–electron scattering more or less counters the drop in mobility due to enhanced phonon emission, leaving the mobility roughly constant.

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